Speed control of two inertia system with servo motor

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December 2, 2010

1 Experimental setup

The control target is mainly composed of a servomotor and a inertia-load disc connected with pulleys and belt. The servo motor has a rotary encoder whose output pulse signal is processed by a counter to generate rotational position. Rotational speed is approximately measured by evaluating difference of rotational pulses. The driving torque of the servo motor is specified by analog voltage generated by D/A converter. Controllers are implemented as a real-time task of RT-Linux on PC.

Table 1: Experimental equipments

Driving motor	YASUKAWA ELEC. SGMAV-02A, SGDV-1R6A 0.088×10^{-4}kgm^2, 200W, 0.637Nm(rated), 1.9Nm(max), 65536ppr
PC	DELL Dimension 2100 (Celeron 1000MHz) RT-Linux 3.2 / Fedora core 1 (kernel 2.4.22)
D/A	CONTEC DA12-4(PCI) (12bit, 10μs)
Counter	CONTEC CNT24-4(PCI)H (24bit, 1MHz)

2 Derivation of plant model

Consider a two inertia system dipicted in Fig. 1.

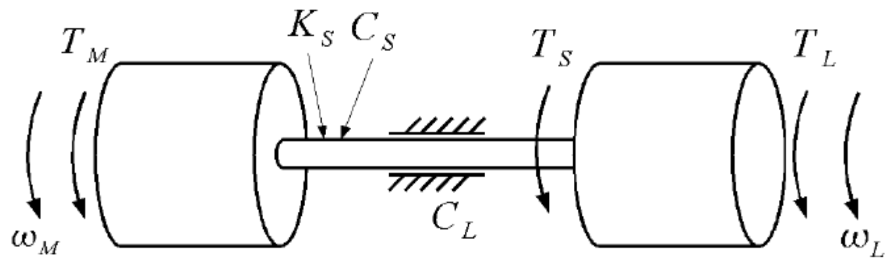

Figure 1: Two Inertia System

Denote variables as following:
θ_M : Rotational angle of driving motor (rad)
ω_M : Rotational speed of driving motor (rad/s)
J_M : Moment of inertia of driving motor (Kg m^2)
T_M : Driving torque (Nm)
θ_L : Rotational angle of load (rad)
ω_L : Rotational speed of load (rad/s)
J_L : Moment of inertia of load (Kg m^2)
T_L : Disturbance torque of load (Nm)
C_L : Damping coefficient due to friction on load
K_S : Torsional stiffness of shaft (Nm/rad)
C_S : Torsional damping coefficient of shaft
T_S : Torsional torque on shaft (Nm)

Let θ_r be a relative angle of the motor and the load as $\theta_r := \theta_M - \theta_L$, then equation of motion of the system is given as following:

$$\begin{aligned} J_M \dot{\omega}_M &= T_M - T_S & (1) \\ T_S &= K_S \theta_r + C_S(\omega_M - \omega_L) & (2) \\ J_L \dot{\omega}_L &= T_L + T_S - C_L \omega_L & (3) \end{aligned}$$

Let state-space variable x be

$$x := \begin{bmatrix} \theta_r \\ \omega_M \\ \omega_L \end{bmatrix}. \tag{4}$$

State-space representation of the system from T_M to ω_M is given by

$$\frac{d}{dt}\begin{bmatrix} \theta_r \\ \omega_M \\ \omega_L \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -\frac{K_S}{J_M} & -\frac{C_S}{J_M} & \frac{C_S}{J_M} \\ \frac{K_S}{J_L} & \frac{C_S}{J_L} & -\frac{C_S+C_L}{J_L} \end{bmatrix}\begin{bmatrix} \theta_r \\ \omega_M \\ \omega_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_M} \\ 0 \end{bmatrix} T_M \tag{5}$$

$$\omega_M = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \theta_r \\ \omega_M \\ \omega_L \end{bmatrix} \tag{6}$$

Torsional resonance frequency f_r and anti-resonance frequency f_a are given respectively as follows:

$$f_r = \frac{1}{2\pi}\sqrt{K_S\left(\frac{1}{J_L} + \frac{1}{J_M}\right)}, \quad f_a = \frac{1}{2\pi}\sqrt{\frac{K_S}{J_L}} \tag{7}$$